Izhikevich Model of a Spiking Neuron: A Qualitative Analysis Through the Phase Plane

Hana Kollárová, BSc.

Department of Applied Informatics, Faculty of Mathematics, Physics, and Informatics, Comenius University of Bratislava
KAI FMFI UK, Mlynska dolina, 842 48 Bratislava
kollarova223@uniba.sk

Abstract

This paper discusses the biological basis for computational models, and introduces the Izhikevich model, which is a derivative of the Hodgkin and Huxley model simplifying the Hodgkin-Huxley's four equations into two. Parametric sensitivity of the Izhikevich model will be explored through input current alterations. Specifically, experiments with different values for stable external current, linearly rising external current, and single- or double pulsed currents were performed. The results qualitatively correlate with the behaviour of biological neurons, proving the Izhikevich model is able to replicate their characteristics appropriately.

1 Introduction

Computational neuroscience uses mathematical models to study how neurons generate and transmit electrical activity. Because the underlying biophysical mechanisms involve many interacting processes, models are typically simplified, with the level of abstraction chosen based on the problem being studied (Cessac and Samuelides (2007)). Detailed conductance-based models can reproduce ionic dynamics with high accuracy, but their complexity limits their use in large-scale simulations and systematic analysis. For this reason, reduced models are commonly used when the focus is on qualitative behaviour such as spike generation and pattern formation.

A widely used example is the Izhikevich neuron model, introduced by Eugene M. Izhikevich (Izhikevich, 2003), which combines low computational cost with the ability to reproduce multiple firing patterns observed in biological neurons. The model is defined as a two-dimensional system:

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I, \tag{1}$$

$$\frac{du}{dt} = a(bv - u), \tag{2}$$

with the after-spike reset:

$$\text{if } v \geq 30 \text{ mV, then } \quad \begin{cases} v \leftarrow c, \\ u \leftarrow u + d. \end{cases} \tag{3}$$

In this formulation, v represents the membrane potential and u is a recovery variable capturing slower processes such as ionic currents and membrane adaptation. The external input I drives the system, while the parameters a, b, c, and d determine the qualitative firing regime. Unlike more detailed conductance-based approaches, these parameters are not tied to specific ion channels but instead control the system's dynamical response.

Compared to the Hodgkin–Huxley model (Izhikevich, 2007), which explicitly models ion channel kinetics using four coupled nonlinear equations, the Izhikevich model reduces the system to two dimensions while retaining key behaviours such as threshold dynamics, spike generation, and refractory effects. This reduction makes it possible to explore how qualitative changes in input affect neuronal activity without the computational overhead of biophysical detail.

Reduced models become particularly relevant when extending the analysis beyond single neurons. In network settings, even simple neuron models can produce complex collective dynamics, and the cost of simulation quickly becomes a limiting factor. In this context, the Izhikevich model provides a practical compromise: it is simple enough for large-scale simulations, but still expressive enough to capture distinct firing patterns.

This paper examines how the behaviour of the Izhikevich model changes under different forms of external input. The analysis focuses on how variations in the input current I influence spike generation and overall system dynamics. Specifically, constant, time-dependent, and pulsed inputs are considered, with the goal of relating observed behaviours to the underlying dynamical structure of the model.

2 Method

In this experiment, the Izhikevich model's stability and dynamics will be explored by manipulating the external current I_{ext}. The model parameters will be kept constant throughout all experiments, taken from Kollárová (2025). Specifically, the biologically-optimized parameter set will be used: $a = 0.01877$, $b = 0.26801$, $c = -66.3083$, $d = 12.4662$.

The initial condition for the system is set to:

$$v_0 = -59.15, \quad u_0 = 0.26801 \cdot (-59.15),$$

which places the system near the so-called "ghost equilibrium". For this parameter regime, the system does not exhibit a stable resting fixed point.

For qualitative analysis, the nullclines of the system are derived directly from the parametrised equations. The $\dot{v}$ nullcline is given by:

$$u = 0.04v^2 + 5v + 140 + I_{\text{ext}}, \tag{4}$$

which forms a quadratic curve in the (v, u) phase plane. The $\dot{u}$ nullcline follows as:

$$u = 0.26801v, \tag{5}$$

which is linear with slope 0.26801. The intersection of these nullclines determines the fixed points of the system. However, under the chosen parameters, the system does not settle into a stable equilibrium and instead exhibits spike-driven dynamics governed by the reset condition.

There will be three parts to the experiment. In the first part, the external current will be kept constant. In the second part, the external current will be rising linearly, and in the third, it will be limited to a specific time interval $t_0 - t_1$, and, therefore, will be a pulse. There will be either exactly one or two pulses at any given time, and they will be of a set external current. In terms of units, the current is expressed in the same abstract units as in Izhikevich (2003). These values are scaled inputs directly added to the voltage dynamics and therefore do not correspond to a physical current unless a specific calibration is introduced.

The external current will vary from 0 to 50 for the stable external current. For the linearly rising current, it will also go from 0 to 50. For the time-dependent external current, it will be set stably at 11.76, and the pulse time will differ.

The pulse intervals for a singular pulse will be set at a time [9,10], [10.11], [13,14], [14,15], and for double pulses at times $\{[30, 31], [32, 33]\}$, $\{[30, 31], [33, 34]\}$, $\{[30, 31], [35, 36]\}$ and $\{[30, 31], [37, 38]\}$. One pulse corresponds to roughly 5 milliseconds.

3 Results

3.1 Results Overview of the Izhikevich Model

The Izhikevich model was systematically evaluated under constant, linearly varying, and pulsed external current inputs in order to assess its ability to reproduce key biological neuronal properties. Across all regimes, the model consistently demonstrated classical excitability, including sub-threshold oscillations, spike generation, spike trains, and refractory behaviour.

3.1.1 Constant External Current

For constant external current, the system showed a clear transition between resting-like behaviour and repetitive firing. At low input levels, only small sub-threshold fluctuations around the resting state were observed. With increasing current, the system crossed a firing threshold and entered a stable limit cycle regime. In this regime, sustained spike trains were produced, confirming tonic spiking behaviour. The transition from no firing to repetitive firing demonstrated the presence of a well-defined excitability threshold, consistent with biological neurons.

For very high constant input, the system exhibited depolarisation without full recovery dynamics, indicating a breakdown of normal spiking structure. This regime corresponds to an excitation-dominated state where the neuron remains in a persistently elevated voltage level. This confirms that excessive stimulation suppresses normal action potential structure, consistent with excitation block-like behaviour.

3.1.2 Linearly Increasing Current

For linearly increasing external current, the system showed a clear range of input values in which spiking behaviour was possible. Below this range, only sub-threshold oscillations were present. Within the intermediate regime, regular spike trains emerged. Above the upper bound, the system no longer produced structured action potentials. This confirms that the model exhibits a bounded excitability window, where biological firing is only possible within a finite range of input strengths.

3.1.3 Single and Double Pulsed Current

For pulsed stimulation, the model demonstrated strong state dependence. A single pulse of sufficient amplitude was able to generate an action potential only when applied at specific phases of the trajectory. When the pulse timing was not aligned with a responsive state of the system, only sub-threshold responses occurred. This confirms that spike generation depends not only on input magnitude, but also on the internal state of the neuron at the time of stimulation.

For double pulse stimulation, refractory effects were clearly observed. Closely spaced pulses resulted in a reduced or absent second spike, while sufficiently separated pulses produced two distinct action potentials. This demonstrates the presence of both relative and absolute refractory dynamics, where excitability is temporarily suppressed following a spike.

3.2 Example: Single Pulse

In addition to constant stimulation, the system was tested under pulsed external current. A single pulse of amplitude $I_{ext} = 11.76$ was applied at times $[9, 10]$. The pulse resulted in a full action potential, as the system was able to cross the effective firing threshold in phase space.

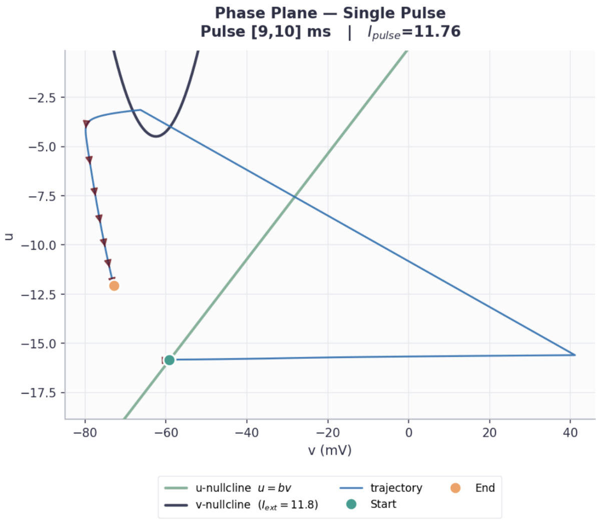

Fig. 1: Phase plane plot of a single pulsed current at time 9-10.

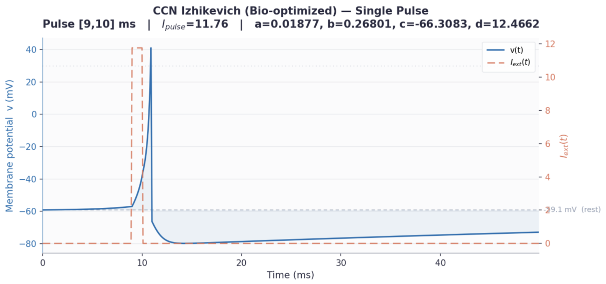

Fig. 2: Resulting voltage curve of the pulsed current at time 9-10.

4 Conclusion

Overall, the model reproduces a wide range of biologically relevant neuronal behaviours, including excitability thresholds, tonic spiking, bounded firing regimes, state-dependent responsiveness, and refractory effects. Each of these properties is confirmed under a distinct stimulation protocol, indicating that the model captures both steady-state and transient neuronal dynamics in a unified framework.

The amplitude and shape of the action potentials were dependent on both the magnitude of the input and the internal state of the system at the time of stimulation. The model reproduces key biological properties of neuronal dynamics, including excitability thresholds, tonic firing under constant drive, and refractory behaviour under repeated stimulation. Even though the variables are dimensionless, the qualitative correspondence to biological neuron behaviour remains strong. The model is therefore suitable for studying input-dependent spiking behaviour in a reduced dynamical system framework.

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